HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 12

MATHEMATICS

2011

HSC COURSE

ASSESSMENT TASK 3

Examiners ~ J. Dillon, S. Gee and P. Biczo

General Instructions

- Reading Time – 3 minutes.
- Working Time – 40 Minutes.
- Attempt ALL questions.
- All Questions are of equal value.
- All necessary working should be shown in each question.
- This paper contains Three (3) questions.
- Total Marks – 30 marks
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators and Mathematical Templates may be used.
- A Table of Standard Integrals is provided for your use.
- Each question is to be started in a new booklet.
- This assessment task must NOT be removed from the Examination Room.

Student Name: ______________________________

Teacher: ______________________________

Question 1 **Start a new booklet** **Marks**

(a) Find

(i) $\int (3x+2)\ dx$ **1**

(ii) $\int (2x+1)^{\frac{1}{2}}\ dx$ **1**

(b) Evaluate

(i) $\int_{1}^{2} (2x-5)(x^2-1)\ dx$ **3**

(ii) $\int_{-3}^{3} \left(x^2+\frac{1}{x^2}\right)\ dx$ **2**

(c) (i) On the same diagram sketch the curves $y=x^2$ and $y=2x-x^2$. **1**

(ii) Show the points of intersection of the curves $y=x^2$ and $y=2x-x^2$ are $(0,0)$ and $(1,1)$. **1**

(iii) Find the area between the curves in the domain $0 \le x \le 1$. **1**

Question 2 continues on the next page .....

Question 2 **Start a new booklet** **Marks**

(a) (i) Consider the function $y = \sqrt{9 - x^2}$. **1**

Copy the following table into your answer booklet.

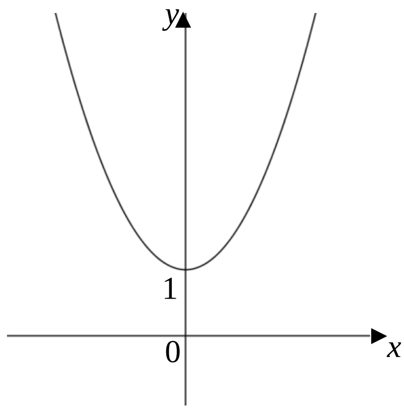
Complete the table, giving answer correct to two decimal places.

x	1	1.5	2
y	2.83		2.24

(ii) Hence, find an approximation for $\int_1^2 \sqrt{9 - x^2}\, dx$ using the Trapezoidal Rule with 2 strips. Give your answer correct to two decimal places. **2**

(b) Use Simpson's Rule with five function values to find an approximation for the area under the curve $y = 2\log_{10} x$ between $x = 2$ and $x = 4$. **2**
Give your answer to two decimal places.

(c)



The graph above shows the curve $y = x^2 + 1$.

(i) Copy the diagram and shade the area bounded by the curve, the x-axis and the lines $x = 0$ and $x = 2$ **1**

(ii) Find the volume of the solid of revolution formed when this area is rotated about the x-axis. **2**

(iii) The area bounded by the parabola $y = x^2 + 1$, the y-axis and the line $y = 4$ is rotated about the y-axis. Find the volume of the solid formed. **2**

Question 3 continues on the next page .....

Question 3 Start a new booklet **Marks**

(a) Jerry joins a superannuation fund, investing $\$P$ at the beginning of every year at 9% p.a. compounding annually.

(i) Write an expression for the value of his investment A_1 at the end of the first year **1**

(ii) Write an expression for the value of his investment A_2 at the end of the second year **1**

(iii) Show that, after *n* years, the value of his investment A_n is given by **2**

$$A_n = \frac{109P}{9}\left(1.09^n - 1\right)$$

(iv) If, after 30 years, he wishes to collect $1 000 000, calculate the value of *P* correct to the nearest dollar. **1**

(b) Lisa borrows $20 000 at 3% per quarter reducible interest. She pays the loan off over 5 years by paying quarterly repayments of $\$R$.

Let A_n be the amount of money that Lisa owes after the *n*th repayment.

(i) Write an expression for A_1. **1**

(ii) Show that $A_n = 20000 \times 1.03^n - R\left(1.03^{n-1} + ... + 1.03^2 + 1.03 + 1\right)$ **2**

(iii) Hence, find the value of *R* to the nearest dollar. **2**

End of Exam

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x > a > 0$$

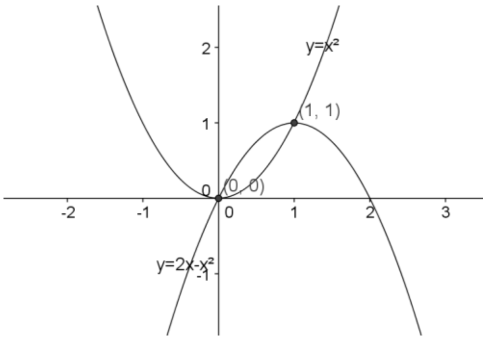
$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Year 12	Mathematics Advanced	Assessment Task 3 2011
Question No. 1	Solutions and Marking Guidelines	

Outcomes Addressed in this Question: P3, P4

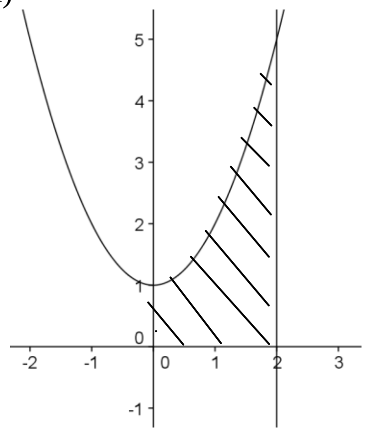
Outcome	Solutions	Marking Guidelines
P3	(a) (i) $\int (3x+2)dx = \frac{3x^2}{2} + 2x + C$ (ii) $(2x+1)^{\frac{1}{2}}dx = \frac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}} + C$ $\int = \frac{(2x+1)^{\frac{3}{2}}}{3} + C$ $Note: \frac{(2x+1)\sqrt{2x+1}}{3} + C$ *is acceptable.*)	**1 mark:** Complete & C **1 mark:** Complete & C
	(b) (i) $\int_1^2 (2x-5)((x^2-1)dx = \int_1^2 (2x^3 - 2x - 5x^2 + 5)dx$ $= \frac{x^4}{2} - x^2 - \frac{5x^3}{3} + 5x \Big\|_1^2$ $= [8 - 4 - \frac{40}{3} + 10] - [\frac{1}{2} - 1 - \frac{5}{3} + 5]$ $= \frac{2}{3} - \frac{17}{6}$ $= \frac{-13}{6}$ *or* -2.17	**3 marks:** Complete solution **2 marks:** Substantial progress **1 mark:** Some progress
	(ii) *Either* $\int_{-3}^{3} x^2 + \frac{1}{x^2}\,dx = \frac{x^3}{3} - \frac{1}{x}\Big\|_{-3}^{3}$ $= [9 - \frac{1}{3}] - [-9 + \frac{1}{3}]$ $= 17\frac{1}{3}$ *Or* $x^2 + \frac{1}{x^2}$ *is an even function* $\int_{-3}^{3} x^2 + \frac{1}{x^2}\,dx = 2\int_0^3 x^2 + \frac{1}{x^2}\,dx$ $= 2\{\frac{x^3}{3} - \frac{1}{x}\}_0^3 =$???????????????	**2 marks:** Complete solution either $17\frac{1}{3}$ or the problem with the question.

Outcome	Solutions	Marking Guidelines
P3, P4	(c) (i) 	**<u>1 mark:</u>** Clear sketch
	(ii) Solving the two equations simultaneously $x^2 = 2x - x^2$ $2x(x-1) = 0$ $x = 0 \; or \; x = 1. \; If \; x = 0, y = 0. \; If \; x = 1, y = 1$ $(0,0) \& (1,1)$	**<u>1 mark:</u>** Show working
	(iii) $Area = \int_0^1 2x - 2x^2 dx$ $= x^2 - \frac{2x^3}{3} \Big]_0^1$ $= 1 - \frac{2}{3}$ $= \frac{1}{3} U^2$	**<u>1 mark:</u>** Find the correct area

Year 12 Task 3	Mathematics	Examination 2011
Question No.2	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

H8 uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines
	a) (i) 2.60	1 mark : correct answer
	(ii) $\int_1^2 \sqrt{9-x^2}\,dx = \frac{0.5}{2}\{2.83+2\times2.60+2.24\}$ $= 2.57$ (to 2 d.p.)	2 marks : correct solution 1 mark: substantial progress towards correct answer
	b) $\int_2^4 2\log_{10}x\,dx = \frac{0.5}{3}\{2\log_{10}2+4\times2\log_{10}2.5+2\log_{10}3\}$ $+ \frac{0.5}{3}\{2\log_{10}3+4\times2\log_{10}3.5+2\log_{10}4\}$ = 1.88 (to 2 d.p.)	2 marks : correct solution 1 mark: substantial progress towards correct answer
	c) (i) 	1 mark: correct area
	(ii) $V=\pi\int_0^2 y^2\,dx = \pi\int_0^2 \left(x^2+1\right)^2 dx$ $=\pi\int_0^2\left(x^4+2x^2+1\right)dx$ $=\pi\left[\frac{x^5}{5}+\frac{2x^3}{3}+x\right]_0^2$ $=\pi\left(\frac{32}{5}+\frac{16}{3}+2\right) = \frac{206\pi}{15}\text{ u}^3$	2 marks : correct solution 1 mark : substantial progress towards correct answer
	(iii) If $y=x^2+1$, then $x^2=y-1$. $V=\pi\int_1^4 x^2\,dy=\pi\int_1^4(y-1)\,dy$ $=\pi\left[\frac{y^2}{2}-y\right]_1^4$ $=\pi\left(8-4-\left(\frac{1}{2}-1\right)\right) = \frac{9\pi}{2}\text{ u}^3$	2 marks : correct solution 1 mark : substantial progress towards correct answer

Outcome Addressed in this Questio
H5 **applies appropriate techniques from the study of** calculus, geometry, probability, trigonometry and **series to solve problems**

Part	Solutions	Marking Guidelines
(a) (i)	$A_1 = P(1.09)$	**Award 1** for correct solution
(ii)	$A_2 = A_1(1.09) + P(1.09)$ $= P(1.09)^2 + P(1.09)$	**Award 1** for correct solution
(iii)	$A_3 = A_2(1.09) + P(1.09)$ $= P\left((1.09)^2 + (1.09)\right) + P(1.09)$ $= P(1.09)^3 + P(1.09)^2 + P(1.09)$ $= P\left(1.09^3 + 1.09^2 + 1.09\right)$ . . . $A_n = P\left(1.09^n + 1.09^{n-1} + \ldots.. + 1.09\right)$ $= P\left(1.09 + 1.09^2 + \ldots.. + 1.09^n\right)$ $= P \times \dfrac{1.09\left(1.09^n - 1\right)}{1.09 - 1}$ $= \dfrac{109P}{9}\left(1.09^n - 1\right)$	**Award 2** for correct solution **Award 1** for substantial progress towards solution.
(iv)	$1000000 = \dfrac{109P}{9}\left(1.09^{30} - 1\right)$ $\therefore P = \dfrac{9}{109} \times \dfrac{1000000}{\left(1.09^{30} - 1\right)}$ $= 6730.597606$ $= \$6731$ (to nearest dollar)	**Award 1** for correct solution
(b) (i)	$A_1 = 20000(1.03) - R$	**Award 1** for correct solution
(ii)	$A_2 = A_1(1.03) - R$ $= 20000(1.03)^2 - 1.03R - R$ $A_3 = A_2(1.03) - R$ $= 20000(1.03)^3 - 1.03^2 R - 1.03R - R$ . . . $A_n = A_{n-1}(1.03) - R$ $= 20000(1.03)^n - 1.03^{n-1}R - 1.03^{n-2}R - \ldots... - R$ $= 20000(1.03)^n - R\left(1.03^{n-1} + 1.03^{n-2} + \ldots.. + 1.03^2 + 1.03\right)$	**Award 2** for correct solution **Award 1** for substantial progress towards solution

(iii)	$A_n = 20000(1.03)^n - R(1.03^{n-1} + 1.03^{n-2} + ... + 1.03^2 + 1.03)$ $= 20000(1.03)^n - R(1.03 + 1.03^2 + ... + 1.03^{n-2} + 1.03^{n-1})$ $= 20000(1.03)^n - R\frac{1(1.03^n - 1)}{1.03 - 1}$ $= 20000(1.03)^n - \frac{100}{3}R(1.03^n - 1)$ $A_{20} = 0$ $0 = 20000(1.03)^{20} - \frac{100}{3}R(1.03^{20} - 1)$ $R = 1344.314152$ $\therefore R = \$1344$ (nearest dollar)	**Award 2** for correct solution **Award 1** for substantial progress towards solution